On assessment of the stochastic errors of radio source position catalogues

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Abstract. *Assessment of the stochastic errors of the radio source position catalogues derived from Very Long Baseline Interferometry (VLBI) observations is important for estimating the quality of the catalogs and their weighting during combination. One of the widely used method for estimation of the catalog stochastic errors is the 3-cotnered hat technique. A critical point of this method is proper accounting for correlations between the compared catalogs. In this poster, we present a new approach to this task.*

Introduction

So called "3-cornered hat" method (3CH) was originally developed for estimation of the stability of frequency standards (Gray and Allan, 1974). It was then applied for investigation of the noise level of various data, in particular, astronomical and geodetic time series and radio source position catalogs. However, despite this method is widely used, its application is not straightforward because it requires a reliable estimate of the correlation between series under investigation. Neglecting correlations often produces unacceptable results, like negative variances. In this work, we investigate some a new possibility to estimate correlations between radio source position catalogs (RSC).

3-cornered hat method

In original formulation, the 3CH method is applied to three series of measurements, which allows us to write the following system of three equations for the pair differences between the series supposing they are uncorrelated:

$$\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2\,, \quad \sigma_{13}^2 = \sigma_1^2 + \sigma_3^2\,, \quad \sigma_{23}^2 = \sigma_2^2 + \sigma_3^2\,.$$

with solution

$$\sigma_1^2 = (\sigma_{12}^2 + \sigma_{13}^2 - \sigma_{23}^2)/2\,, \quad \sigma_2^2 = (\sigma_{12}^2 + \sigma_{23}^2 - \sigma_{13}^2)/2\,, \quad \sigma_3^2 = (\sigma_{13}^2 + \sigma_{23}^2 - \sigma_{12}^2)/2\,.$$

For arbitrary number of measurement series one can use the following solution derived by Barnes (1992).

$$\sigma_i^2 = \frac{1}{M-2}\left(\sum_{j=1}^{M} \sigma_{ij}^2 - B\right)\,,$$

$$B = \frac{1}{2(M-1)} \sum_{k=1}^{M} \sum_{j=1}^{M} \sigma_{kj}^2\,,$$

$$\sigma_{ii} = 0, \sigma_{ij} = \sigma_{ji}.$$

With correlations, the system to be solve consists of the equations: $\sigma_{ij}^2 = \sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j\,.$

The key point is to obtain a reliable estimates of ρ_{ij}.

Application to RSC

Several developments in using the 3CH for RSC made since the 1990s in the Main Astronomical Observatory (MAO) NANU, Ukraine (Molotaj et al. 1998, Bolotin & Lytvyn 2010). Several 3CH modifications method was used, all based on analysis of differences between the pairs of input RSCs and combined one. The authors discussed some shortcomings of this approach. In this presentation, we extended MAO method for using all the RSCs simultaneously. For this purpose we computed the correlations between each pair of catalogs and all others supposing that ρ_{ij} can be approximated by the correlation between catalog differences *Corr(Δij,Δik)*. results of this computation are shown in Table 1. One can see some features of the correlations.

− Correlation in RA and DE are very similar, which confirms results of other authors.

− Discrepancies between the columns show discrepancies between corresponding catalogs confirmed by the WRMS.

− There is no clear dependence on the software used.

Table 1. Correlations between RSC differences *Corr(Δij,Δik)* approaching correlation between *i*-th and *j*-th catalogs shown in the first column; the next 7 columns corresponds to *k*-th catalog.

Catalogs	AUS	BKG	GSF	IAA	MAO	OPA	USN	Mean
AUS BKG			0.0259 -.0807	0.1870 0.1533	0.1199 0.0604	0.0057 -.1081	0.0159 0.1137	0.0709 0.0277
AUS GSF		-.1868 -.2932		0.2932 0.3056	0.1493 0.1381	-.0173 -.1742	0.0302 0.1392	0.0537 0.0231
AUS IAA		-.1723 -.1807	-.0909 0.0087		0.1013 0.0346	0.0414 -.0723	0.0533 0.1498	-.0134 -.0120
AUS MAO		-.1934 -.2939	-.1408 -.1606	-.3258 -.3361		0.0470 0.0500	0.0709 0.2325	-.1084 -.1016
AUS OPA		-.2004 -.3461	-.1317 -.3034	-.3114 -.3820	-.1720 -.2157		0.0565 0.2801	-.1518 -.1934
AUS USN		-.2059 -.2208	-.1595 -.0670	-.3318 -.2261	-.1929 -.1303	-.0722 0.0389		-.1925 -.1211
BKG GSF	0.8861 0.9473			0.6182 0.6901	0.7319 0.7355	0.2891 0.4078	0.5264 0.6137	0.6103 0.6789
BKG IAA	0.6918 0.8501		-.2146 -.2247		0.5439 0.5640	0.2419 0.3013	0.3826 0.6202	0.3291 0.4222
BKG MAO	0.9338 0.9182		-.0243 -.0233	-.4356 -.3848		0.0391 0.1063	0.3737 0.5263	0.1773 0.2285
BKG OPA	0.9553 0.9503		-.0305 0.0832	-.5838 -.6311	-.7074 -.6825		0.5047 0.6572	0.0277 0.0754
BKG USN	0.9546 0.9278		-.2769 0.0039	-.5805 -.4490	-.7059 -.5252	-.3158 -.0821		-.1849 -.0249
GSF IAA	0.8597 0.9131	0.5391 0.5375			0.5383 0.5779	0.1396 0.2329	0.4398 0.5475	0.5033 0.5618
GSF MAO	0.8623 0.9239	0.6230 0.6336		-.7176 -.6057		0.0242 0.1718	0.6252 0.5734	0.2834 0.3394
GSF OPA	0.8373 0.9452	0.9463 0.9414		-.9694 -.9484	-.9470 -.9336		0.9209 0.7672	0.1576 0.1544
GSF USN	0.8316 0.9255	0.6626 0.6911		-.8019 -.6795	-.8209 -.7137	-.2426 -.1620		-.0742 0.0123
IAA MAO	0.6505 0.8170	0.4284 0.5258	0.1595 0.2652			0.1563 0.2925	0.4145 0.6425	0.3618 0.5086
IAA OPA	0.5894 0.7985	0.5490 0.5379	0.1041 0.0826		-.5286 -.5622		0.4546 0.6848	0.2337 0.3083
IAA USN	0.5863 0.7879	0.4369 0.4039	0.1708 0.0351		-.5239 -.4554	-.1777 -.0732		0.0985 0.1396
MAO OPA	0.9584 0.9535	0.6547 0.6377	0.2196 0.1491	0.7408 0.6094			0.6601 0.7115	0.6467 0.6122
MAO USN	0.9499 0.9236	0.3558 0.4239	-.0876 -.0393	0.5303 0.3668		0.1123 0.0297		0.3721 0.3409
OPA USN	0.9904 0.9679	0.6570 0.6950	0.1234 0.2910	0.7943 0.6641	0.8014 0.7044			0.6733 0.6645

Conclusion

Proposed method of computation of the correlation between RSCs can provide a reasonable estimates of ρ_{ij} in a case of sufficiently large number of compared catalogs.

Supplement investigations are needed, in particular, of the impact of the large-scale systematic differences between RSCs

References

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