The re-definition of the astronomical unit of length: reasons and consequences

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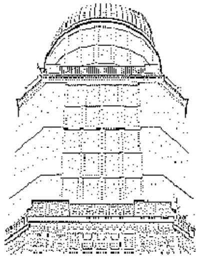

What is the astronomical unit?

Traditionally it has been considered to be a measure of the mean distance of the Earth from the Sun.

Originally it was regarded as the length of the semi-major axis of the Earth's orbit.

The practice of using the value of the Gaussian constant, *k* (Gauss, 1809) as a fixed constant which served to define the au has been in use, unofficially since the 19 th century and officially since 1938.

The "old" definition

- The astronomical unit of length is such that the Newtonian gravitational constant G is equal to the square of the Gaussian constant k

$$G = k^2 = 0.000\,295\,912\,208\,285\,591\,102\,5$$

provided that the mass of the Sun and the day (86400 SI seconds) are taken as the units of mass and time, respectively.

The Gaussian constant $k = 0.017\,202\,098\,95$ is a defining constant.

- The need for a specific scale unit in planetary orbit analyses was due to the lack of precise distance measurements in the solar system.

- The concept is similar to other cases when a unit is defined indirectly by fixed values of some natural constants (e.g. $G = c = \hbar = 1$ for the geometrized units)

IAU 1976 definition of the astronomical unit

«The astronomical unit of length is that length (A) for which the Gaussian gravitational constant (*k*) takes the value of 0.01720209895 when the units of measurements are the astronomical unit of length, mass and time.

The dimensions of k^2 are those of the constant of gravitation (G), i.e., $L^3M^{-1}T^{-2}$. The term "unit distance" is also for the length A. »

Equivalently (cf. The SI brochure): « The radius of an unperturbed circular Newtonian orbit about the Sun of a particle having infinitesimal mass, moving with an angular frequency of 0.017 202 098 95 radians per day. »

Consequences of the current definition of the astronomical unit

- The scale distance in the solar system is provided by the value, A, of the astronomical unit in metres that is fitted to a planetary ephemeris.

- The length of astronomical unit in Système International (SI) meters depends on the theory of motion and observations being used.

- The value in SI units of GM_S is calculated by the formula: $GM_S = A^3k^2/D^2$

The IAU 1976 System of astronomical constants

Defining constants:		
1.	Gaussian gravitational constant	$k = 0{\cdot}017\,202\,098\,95$
2.	Speed of light	$c = 299\,792\,458\ \mathrm{m\,s^{-1}}$
Primary constants:		
3.	Light-time for unit distance	$\tau_A = 499{\cdot}004\,782\ \mathrm{s}$ [499·004 7838 ...]
4.	Equatorial radius for Earth	$a_e = 6378\,140\ \mathrm{m}$ [6378 137]
5.	Dynamical form-factor for Earth	$J_2 = 0{\cdot}001\,082\,63$ [0·001 082 626]
6.	Geocentric gravitational constant	$GE = 3{\cdot}986\,005 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$ [$3{\cdot}986\,004\,33 \ldots \times 10^{14}$]
7.	Constant of gravitation	$G = 6{\cdot}672 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$
8.	Ratio of mass of Moon to that of Earth	$\mu = 0{\cdot}012\,300\,02$ [0·012 300 038]
9.	General precession in longitude, per Julian century, at standard epoch 2000	$p = 5029''0966$
10.	Obliquity of the ecliptic, at standard epoch 2000	$\epsilon = 23^\circ\,26'\,21''448$
Derived constants:		
11.	Constant of nutation, at standard epoch 2000	$N = 9''2025$
12.	Unit distance	$c\tau_A = A = 1{\cdot}495\,978\,70 \times 10^{11}\ \mathrm{m}$ [$1{\cdot}495\,978\,706\,91 \times 10^{11}$]
13.	Solar parallax	$\arcsin(a_e/A) = \pi_\odot = 8''794\,148$ [8''794 144]
14.	Constant of aberration, for standard epoch 2000	$\kappa = 20''49\,552$
15.	Flattening factor for the Earth	$f = 0{\cdot}003\,352\,81 = 1/298{\cdot}257$
16.	Heliocentric gravitational constant	$A^3k^2/D^2 = GS = 1{\cdot}327\,124\,38 \times 10^{20}\ \mathrm{m^3\,s^{-2}}$ [$1{\cdot}327\,124\,40 \ldots \times 10^{20}$]
17.	Ratio of mass of Sun to that of the Earth	$(GS)/(GE) = S/E = 332\,946{\cdot}0$ [332 946·050 895 ...]
18.	Ratio of mass of Sun to that of Earth + Moon	$(S/E)/(1+\mu) = 328\,900{\cdot}5$ [328 900·561 400]
19.	Mass of the Sun	$(GS)/G = S = 1{\cdot}9891 \times 10^{30}\ \mathrm{kg}$

The IAU 2009 System of astronomical constants

- Table 1

		Auxiliary Defining Constants	
k[a]	Gaussian gravitational constant	$1.720209895 \times 10^{-2}$	[18, 15]
L_G	1−d(TT)/d(TCG)	$6.969290134 \times 10^{-10}$	[19, 32]
L_B	1−d(TDB)/d(TCB)	$1.550519768 \times 10^{-8}$	[20]
TDB_0[b]	TDB−TCB at T_0	-6.55×10^{-5} s	[20]
θ_0[c]	Earth rotation angle at J2000.0	0.7790572732640 revolutions	[19, 5]
$d\theta/\mathrm{dUT1}$[c]	Rate of advance of Earth rotation angle	1.00273781191135448 revolutions UT1-day^{-1}	[19, 5]

Constant	Description	Value	Uncertainty	Reference
		Natural Measurable Constants		
G	Constant of gravitation	6.67428×10^{-11} m^{3}kg^{-1}s^{-2}	6.7×10^{-15} m^{3}kg^{-1}s^{-2}	CODATA, 2006
		Other Constants		
au[d]	Astronomical unit	$1.49597870700 \times 10^{11}$ m	3 m	Pitjeva & Standish 2009
L_C	Average value of 1−d(TCG)/d(TCB)	$1.48082686741 \times 10^{-8}$	2×10^{-17}	
		Body Constants[e]		
GM_S	Heliocentric gravitational constant	$1.32712442099 \times 10^{20}$ m^{3}s^{-2} (TCB-compatible) $1.32712440041 \times 10^{20}$ m^{3}s^{-2} (TDB-compatible)	1.0×10^{10} m^{3}s^{-2} (TCB-compatible) 1.0×10^{10} m^{3}s^{-2} (TDB-compatible)	Folkner et al. 2008, from the ua fitted to DE 421

The IAU 2009 System of astronomical constants - definitions and values

- The IAU 2009 System of astronomical constants (IAU 2009 Resolution B2) retains the IAU 1976 definition of the astronomical unit.

- The TDB-compatible value of the astronomical unit (Table 1) is an average (Pitjeva and Standish 2009) of recent estimates for the astronomical unit defined by *k.*

- The TDB-compatible value for GM_S (Table 1) is consistent with the value of Table 1 for the astronomical unit to within the errors of the estimate.

Modern context

- There is a need for a self-consistent set of units and numerical standards for use in modern dynamical astronomy in the framework of General Relativity,

- the accuracy of modern range measurements makes the use of relative distances unnecessary: BCRS distances between solar system bodies are now known very well so that there is no need to decouple angular positions and distances,

- modern planetary ephemerides (INPOP08, DE423, EPM2008) can provide GM_S directly in SI units and even trace the expected time-dependence of this quantity,

- extending the current definition of the astronomical unit to the relativistic framework would require several additional conventions making the definition even more complicated.

Defining the astronomical unit as a fixed number of SI metres

1. Proposal

- The astronomical unit be re-defined to be a conventional unit of length,
- astronomical unit = L_A metres exactly, L_A being a defining number,
- The defining number should be, for continuity reason, the value for the current best estimate of the ua in m as adopted by IAU 2009 Resolution B2

The CCU (of the CIPM) declared its support to move to a fixed relationship to the SI metre through a defining number

Defining the astronomical unit as a fixed number of SI metres

2. Consequences

- *k* will not have a role any more; it should be deleted from the IAU System of astronomical constants,
- the experimental determination of the ua in SI unit, will be abandoned,
- GM_{Sun} will be determined experimentally,
- the ua would limit its role to that of a unit of length of "convenient" size for some applications.

The IAU 2012 Resolution proposal

- the astronomical unit be re-defined to be a conventional unit of length equal to 149 597 870 700 m exactly, as adopted in IAU 2009 Resolution B2

- this definition be used with all time scales (such as TCB, TDB, TCG, TT, etc.),

- the Gaussian gravitational constant *k* be deleted from the system of astronomical constants,

- the value of the heliocentric gravitation constant, GM_S, be determined observationally in SI units,

- the unique symbol au be used for the astronomical unit.

Defining the astronomical unit as a fixed number of SI metres

3. The advantages with respect to the old definition

- ✓ Eliminates deviation from SI
- ✓ Eliminates dependence of the unit on theories of motion on theories of motion
- ✓ Provides a self-consistent set of units in the relativistic framework
- ✓ Avoids time-dependent units if time variation of solar mass is considered
- ✓ Permits direct determination of time variation in solar mass parameter GM_{Sun} in SI units

Defining the astronomical unit as a fixed number of SI metres

4. Impacts of the change

- Mainly concerns those in the field of high-accuracy solar system dynamics.

- Although astronomical unit defines parsec and thus the whole astronomical distance ladder, the relative difference between the old and the new definitions will not exceed 10^{-10} , so no significant effect considering relative errors of cosmic distances outside solar system.