Evaluation of the accuracy of the IAU 2006/2000 precession-nutation

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The IAU 2000/2006 definitions and models for precession-nutation

IAU 2000 Resolutions	IAU 2006 Resolutions
Resolution B1.3 *Definition of BCRS and GCRS*	
Resolution B1.6 *IAU 2000 Precession-Nutation Model*	Resolution B1 *Adoption of the P03 Precession and definition of the ecliptic*
Resolution B1.7 *Definition of Celestial Intermediate Pole*	
Resolution B1.8 *Definition and use of CEO and TEO*	Resolution B2 *Harmonization of the names to CIO and TIO*

IAU 2006/2000A Precession-nutation

IAU 2000 (Resolution B1.6)

- **adopted** the IAU2000 precession-nutation *(Mathews, Herring, Buffett, 2002)*
  which was implemented in the IERS Conventions 2003

  IAU 2000A Nutation *(non-rigid Earth model)*

  IAU 2000 Precession = IAU 1976 *(Lieske et al. 1977)* + corrections to precession rates
  $d\psi_A$ (IAU 2000) = −0.299 65”/c ; $d\omega_A$ (IAU 2000) = −0.025 24”/c

  Celestial pole offsets *at J2000 (VLBI estimates)*
  ξ_0 (IAU 2000) = −16.6170 mas ; η_0 (IAU 2000) = −6.8192 mas

IAU 2006 (Resolution B1)

- **adopted** the P03 precession *(Capitaine, Chapront, Wallace, 2003)*
  dynamical model consistent with IAU 2000A nutation and with non-rigid Earth
  which was implemented in the IERS Conventions 2010

- **recommended** improved definitions
  (ecliptic, precession of the equator, precession of the ecliptic)

Main features of the IAU 2000 nutation

MHB nutation for a non-rigid Earth: Rigid Earth nutation (prograde and retrograde amplitudes) * tansfer function

Rigid Earth nutation (Souchay et al. 1999)

Analytical solution providing semi-analytical series: 1365 luni-solar and planetary terms “in-phase” and “out-of-phase” components (amplitudes between 17.2” and 0.1 µas ; periods between 3 d and 101 cy).

transfer function (Mathews et al. 2002)

Derived from the solution of equations obtained by generalization of the *SOS equations* (Sasao et al. 1980) for the variations in rotation of the Earth’s mantle and fluid core, with *Basic Earth Parameters (BEP)* based on *Model for the dynamics of the Earth’s interior and for modeling the dissipative phenomena) and* fitted to VLBI data.

- e, e_f : dynamical ellipticity of the Earth and its fluid core, respectively,
- $\kappa = ek_2/k_s$, γ: compliance parameters representing the deformabilities of the whole Earth and its fluid core, respectively under tidal forcing,
- K^{CMB} and K^{ICB} : core-mantle and outer core to inner core couplings due to the magnetic fields crossing the boundaries of the fluid core,

The scale factor for the precession rate and nutation amplitudes is $S_{MHB} = H_d = e/(1 + e)$.

Basic Earth Parameters	**Estimate**	**Correction to** hydrostatic equilibrium
e_f	0.0026456 ±20	0.0000973
κ	0.0010340 ±92	-0.0000043
γ	0.0019662 ±14	0.0000007
e	0.0032845479 ±12	0.000037
Im $K^{(CMB)}$	-0.0000185 ±14	
Re $K^{(ICB)}$	0.00111 ±10	
Im $K^{(ICB)}$	-0.00078 ±13	
rms residuals	**0.0132 mas**	

MHB BEP estimated from VLBI

(Mathews et al. 2002)

Main features of the IAU 2006 precession

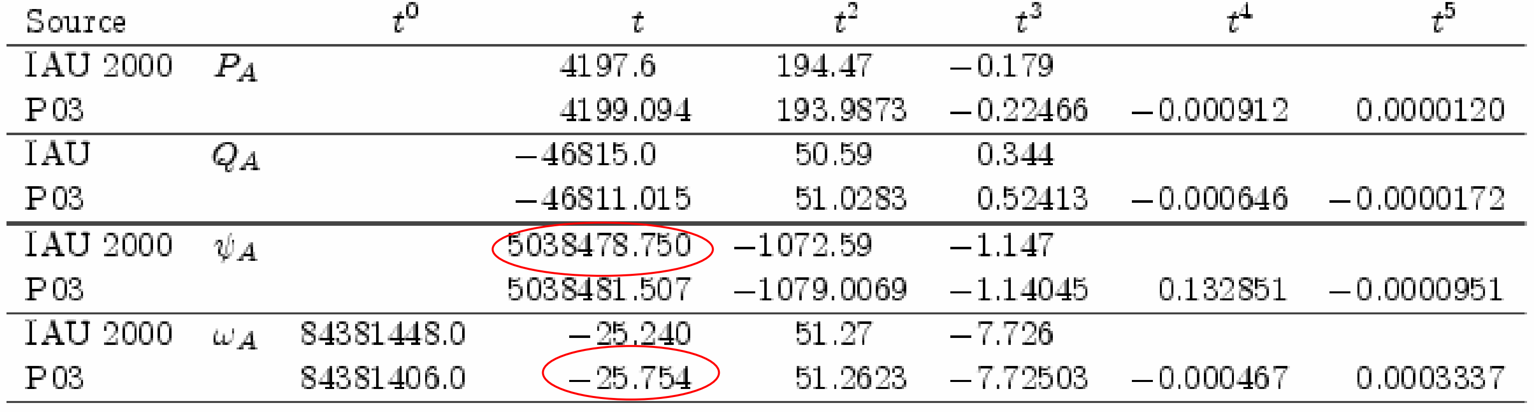
- The IAU 2006 precession provides improved polynomial expressions for both the precession of the ecliptic and the precession of the equator, the latter being consistent with dynamical theory while matching the IAU 2000A precession rate for continuity reasons.

- The precession of the equator was derived from the dynamical equation expressing the motion of the mean pole about the ecliptic pole.

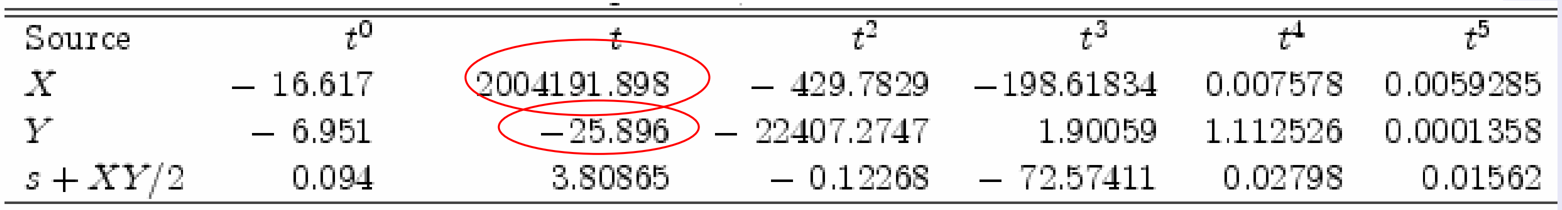
- The solution is based on:
  - the IAU 2000 precession rates in longitude and obliquity,
  - the value, ε_0 = 84381.406'', from Chapront et al. (2002) for the mean obliquity of the ecliptic at J2000.0,
  - contributions to the precession rates r_ψ , r_ε from Williams 1994, Brumberg et al. 1998, Mathews et al. 2002,
  - correction in the precession rate for the change in the J2000 obliquity from IAU2000 to P03,
  - $dJ_2/dt = -3.0 \times 10^{-11}$/ yr

IAU 2006 expressions for precession

(Capitaine et al. 2003)



			mas	mas/cy	mas/cy^2	mas/cy^3	mas/cy^4	mas/cy^5
	Source		t^0	t	t^2	t^3	t^4	t^5
ecliptic	IAU 2000	P_A		4197.6	194.47	−0.179		
	P03			4199.094	193.9873	−0.22466	−0.000912	0.0000120
	IAU	Q_A		−46815.0	50.59	0.344		
	P03			−46811.015	51.0283	0.52413	−0.000646	−0.0000172
equator (equinox based quantities)	IAU 2000	ψ_A		5038478.750	−1072.59	−1.147		
	P03			5038481.507	−1079.0069	−1.14045	0.132851	−0.0000951
	IAU 2000	ω_A	84381448.0	−25.240	51.27	−7.726		
	P03		84381406.0	−25.754	51.2623	−7.72503	−0.000467	0.0003337



	Source	t^0	t	t^2	t^3	t^4	t^5
equator (CIO based quantities)	X	− 16.617	2004191.898	− 429.7829	−198.61834	0.007578	0.0059285
	Y	− 6.951	−25.896	− 22407.2747	1.90059	1.112526	0.0001358
	$s + XY/2$	0.094	3.80865	− 0.12268	− 72.57411	0.02798	0.01562

(ψ_{A1} x sinε, ω_{A1}) ; (X_1, $Y_{1)}$) : components of the precession rates of the equator

The polynomial coefficients for all the precession angles are in Hilton et al. (2006)

IAU 2006/2000A$_{R06}$ expressions for the GCRS coordinates of the CIP

$$
\begin{aligned}
X = {} & -0.''016617 + 2004.''191898\, t - 0.''4297829\, t^2 \\
& - 0.''19861834\, t^3 - 0.''000007578\, t^4 + 0.''0000059285\, t^5 \\
& + \Sigma_i\, [(a_{s,0})_i \sin(\text{ARGUMENT}) + (a_{c,0})_i \cos(\text{ARGUMENT})] \\
& + \Sigma_i\, [(a_{s,1})_i\, t \sin(\text{ARGUMENT}) + (a_{c,1})_i\, t \cos(\text{ARGUMENT})] \\
& + \Sigma_i\, [(a_{s,2})_i\, t^2 \sin(\text{ARGUMENT}) + (a_{c,2})_i\, t^2 \cos(\text{ARGUMENT})] \\
& + \ldots \\
Y = {} & -0.''006951 - 0.''025896\, t - 22.''4072747\, t^2 \\
& + 0.''00190059\, t^3 + 0.''001112526\, t^4 + 0.''0000001358\, t^5 \\
& + \Sigma_i\, [(b_{c,0})_i \cos(\text{ARGUMENT}) + (b_{s,0})_i \sin(\text{ARGUMENT})] \\
& + \Sigma_i\, [(b_{c,1})_i\, t \cos(\text{ARGUMENT}) + (b_{s,1})_i\, t \sin(\text{ARGUMENT})] \\
& + \Sigma_i\, [(b_{c,2})_i\, t^2 \cos(\text{ARGUMENT}) + (b_{s,2})_i\, t^2 \sin(\text{ARGUMENT})] \\
& + \ldots
\end{aligned}
$$

precession; effects of frame biases; nutation; cross terms precession x *nutation*

$(a_{s,1})_i$, $(a_{c'1})_i$, $(b_{c'1})_i$, $(b_{s'1})_i$: take into account the dJ_2/dt contribution

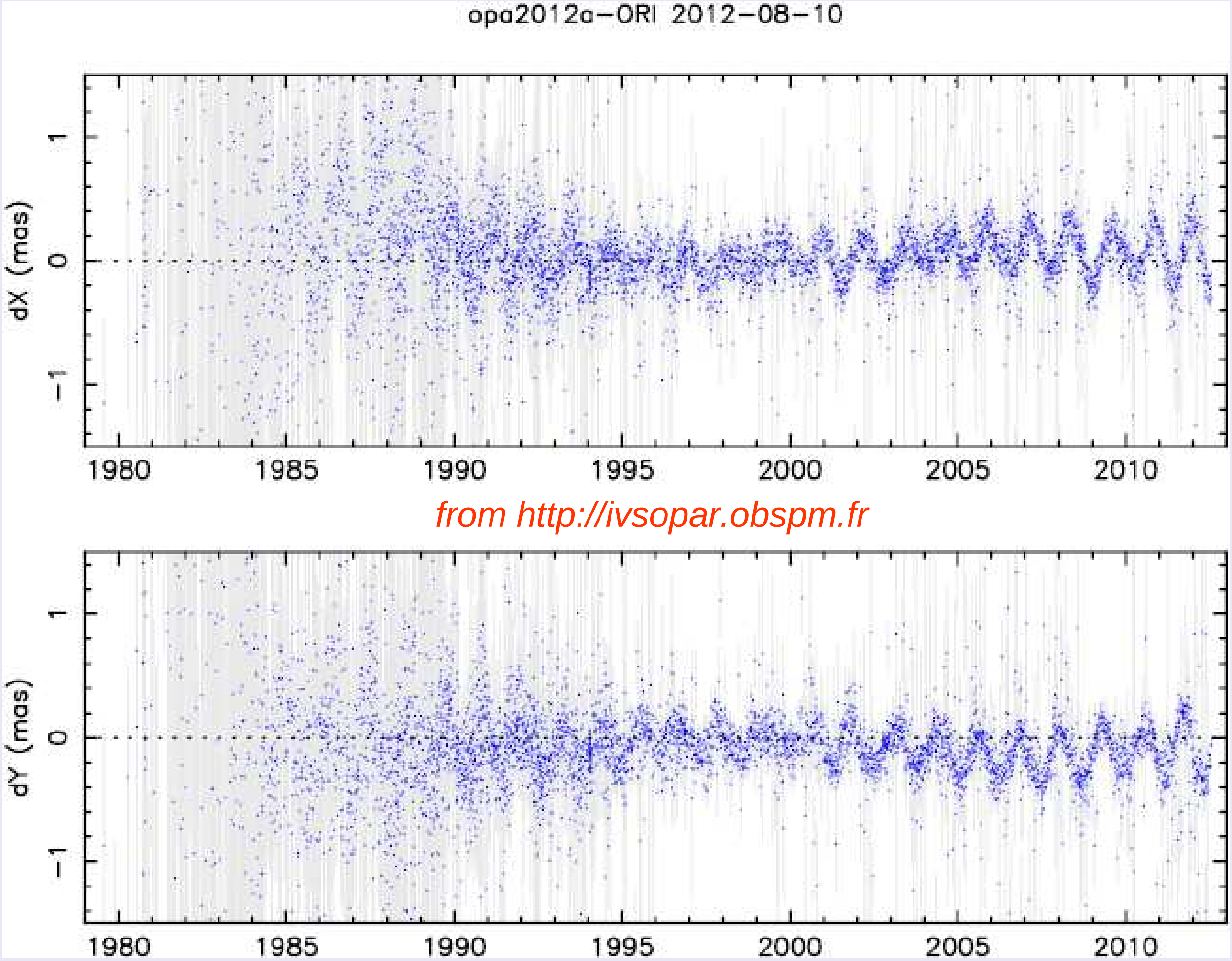
Recently proposed improvements in precession-nutation theory

- Consideration of a relativistic theory of precession and nutation: effect of (i) the post-Newtonian torque, (ii) the geodesic precession as an additional torque (Klioner et al., *Proc. Journées 2007*).
- Influence of the inner core geopotential variations on nutation (Escapa et al., *Proc. Journées 2010*).
- Contribution of the second order torque to precession and nutation (Lambert & Mathews, *A&A 481, 2008).*
- Contribution of the Poisson terms of the tidal potential to nutation (Folgueira et al. *A&A 469, 2007*).
- Contribution of oceanic and atmospheric excitations to nutation (Vondrák & Ron, Proc. Journées 2007).
- Effect of the physical properties of the core-mantle boundary (Koot et al., *Proc Journées 2007* and *2011*).
- Precession expressions for long time intervals (Vondrák et al., *A&A 2011*).

→ *Effects of amplitudes from a tens of μas to a ten of μas in the periodic terms and hundreds of μas/cy for Poisson terms and linear terms*

The observed precession-nutation: VLBI celestial pole offsets

(corrections to the IAU 2006/2000 precession-nutation model)



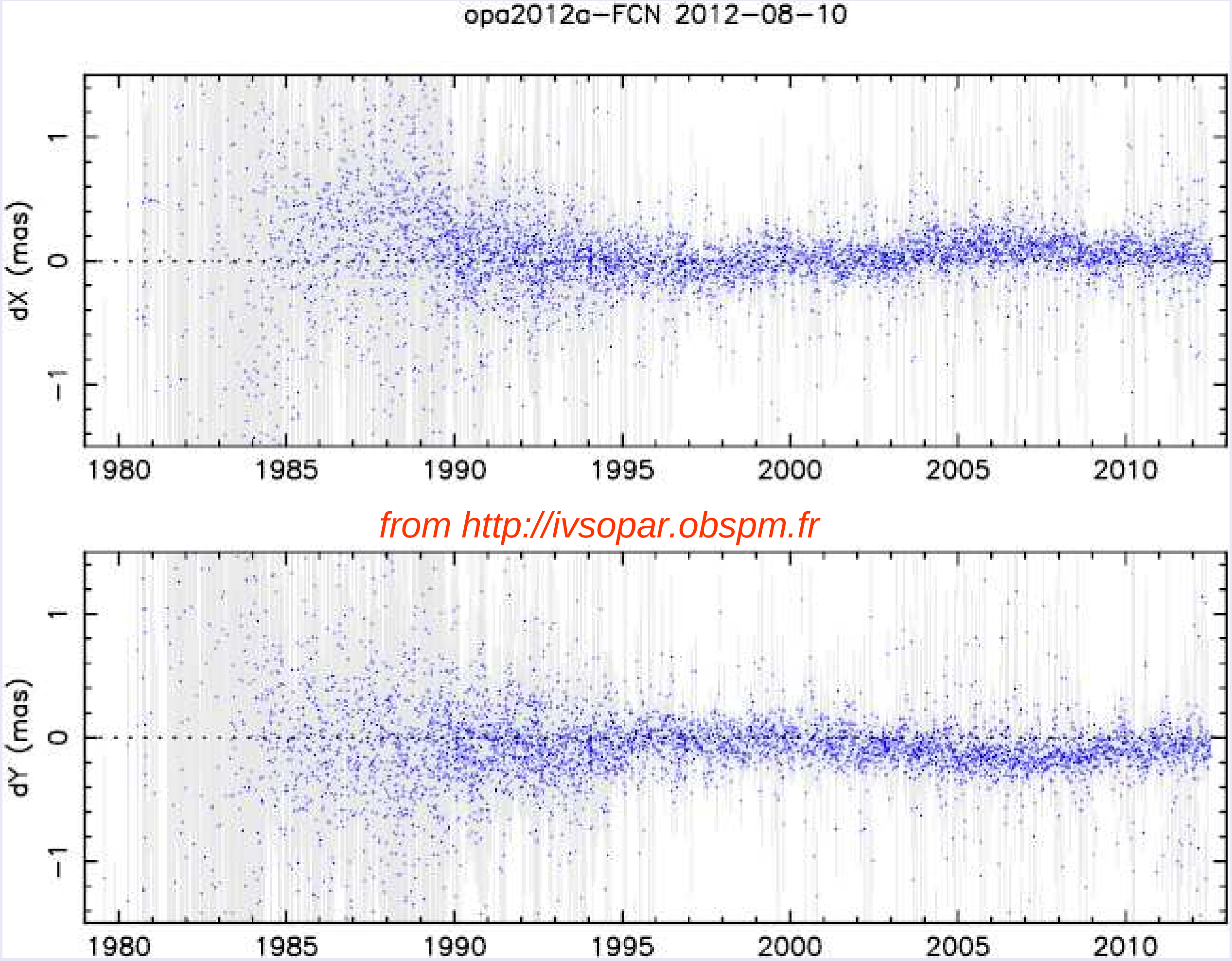
from http://ivsopar.obspm.fr

current uncertainty

~ a few tens of µas
for X, Y

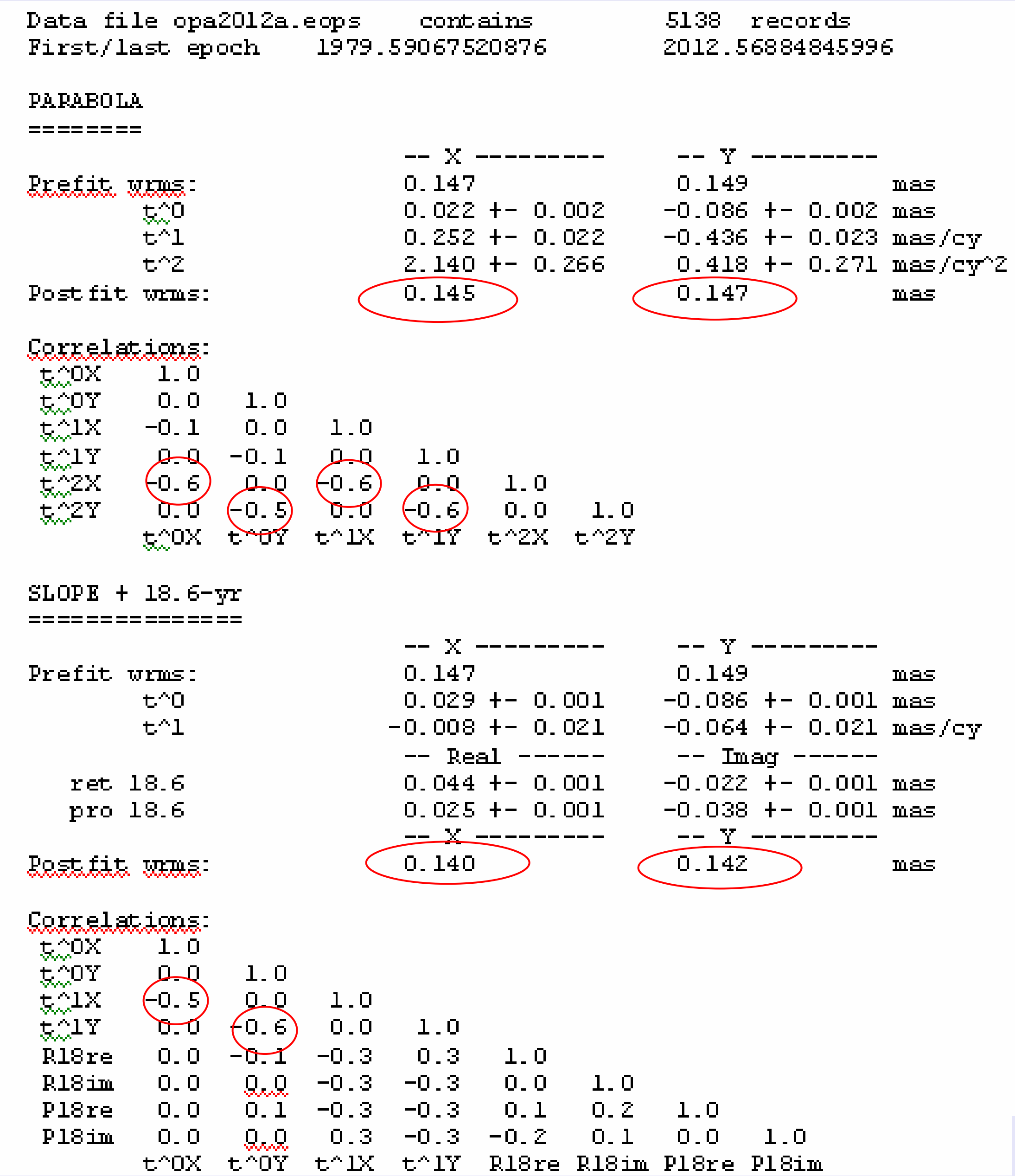
JD7 Space-time reference systems for future research, 28th IAU GA, Beijing, August 2012

VLBI celestial pole offsets (w.r.t. the IAU 2006/2000 precession-nutation) corrected for the FCN



from http://ivsopar.obspm.fr

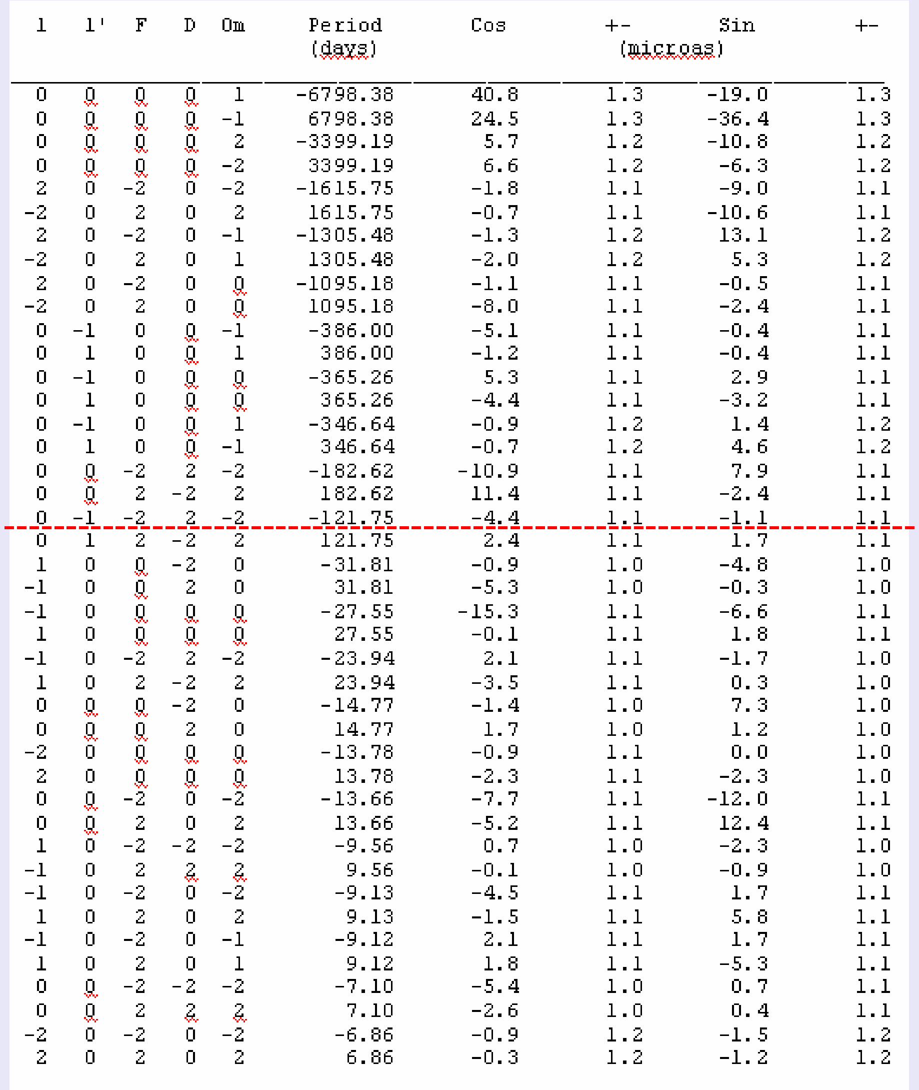
VLBI fit (secular and long period terms)



```
Data file opa2012a.eops    contains         5138  records
First/last epoch     1979.59067520876        2012.56884845996

PARABOLA
========
                            -- X ---------      -- Y ---------
Prefit wrms:                0.147               0.149           mas
        t^0                 0.022 +- 0.002     -0.086 +- 0.002 mas
        t^1                 0.252 +- 0.022     -0.436 +- 0.023 mas/cy
        t^2                 2.140 +- 0.266      0.418 +- 0.271 mas/cy^2
Postfit wrms:               0.145               0.147           mas

Correlations:
 t^0X     1.0
 t^0Y     0.0    1.0
 t^1X    -0.1    0.0    1.0
 t^1Y     0.0   -0.1    0.0    1.0
 t^2X    -0.6    0.0   -0.6    0.0    1.0
 t^2Y     0.0   -0.5    0.0   -0.6    0.0    1.0
         t^0X   t^0Y   t^1X   t^1Y   t^2X   t^2Y

SLOPE + 18.6-yr
===============
                            -- X ---------      -- Y ---------
Prefit wrms:                0.147               0.149           mas
        t^0                 0.029 +- 0.001     -0.086 +- 0.001 mas
        t^1                -0.008 +- 0.021     -0.064 +- 0.021 mas/cy
                            -- Real ------      -- Imag ------
   ret 18.6                 0.044 +- 0.001     -0.022 +- 0.001 mas
   pro 18.6                 0.025 +- 0.001     -0.038 +- 0.001 mas
                            -- X ---------      -- Y ---------
Postfit wrms:               0.140               0.142           mas

Correlations:
 t^0X     1.0
 t^0Y     0.0    1.0
 t^1X    -0.5    0.0    1.0
 t^1Y     0.0   -0.6    0.0    1.0
 R18re    0.0   -0.1   -0.3    0.3    1.0
 R18im    0.0    0.0   -0.3   -0.3    0.0    1.0
 P18re    0.0    0.1   -0.3   -0.3    0.1    0.2    1.0
 P18im    0.0    0.0    0.3   -0.3   -0.2    0.1    0.0    1.0
         t^0X   t^0Y   t^1X   t^1Y   R18re  R18im  P18re  P18im
```

http://ivsopar.obspm.fr

Nutation corrections fit to VLBI



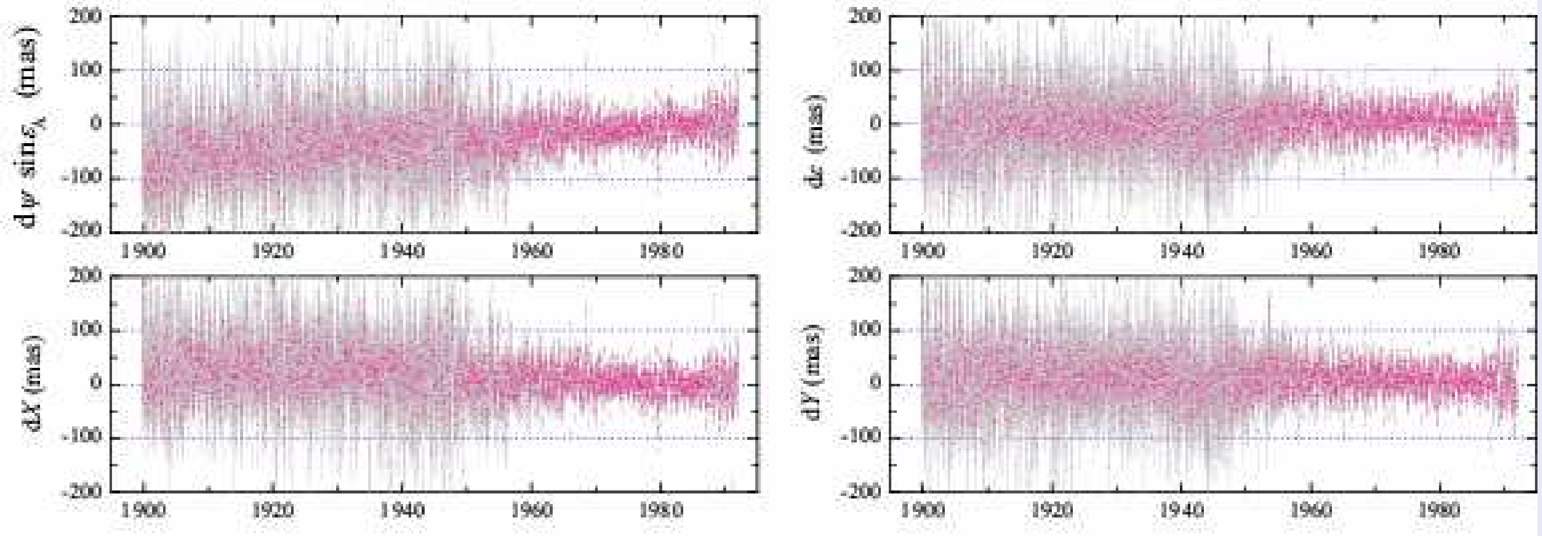
l	l'	F	D	Om	Period (days)	Cos	+- (microas)	Sin	+-
0	0	0	0	1	-6798.38	40.8	1.3	-19.0	1.3
0	0	0	0	-1	6798.38	24.5	1.3	-36.4	1.3
0	0	0	0	2	-3399.19	5.7	1.2	-10.8	1.2
0	0	0	0	-2	3399.19	6.6	1.2	-6.3	1.2
2	0	-2	0	-2	-1615.75	-1.8	1.1	-9.0	1.1
-2	0	2	0	2	1615.75	-0.7	1.1	-10.6	1.1
2	0	-2	0	-1	-1305.48	-1.3	1.2	13.1	1.2
-2	0	2	0	1	1305.48	-2.0	1.2	5.3	1.2
2	0	-2	0	0	-1095.18	-1.1	1.1	-0.5	1.1
-2	0	2	0	0	1095.18	-8.0	1.1	-2.4	1.1
0	-1	0	0	-1	-386.00	-5.1	1.1	-0.4	1.1
0	1	0	0	1	386.00	-1.2	1.1	-0.4	1.1
0	-1	0	0	0	-365.26	5.3	1.1	2.9	1.1
0	1	0	0	0	365.26	-4.4	1.1	-3.2	1.1
0	-1	0	0	1	-346.64	-0.9	1.2	1.4	1.2
0	1	0	0	-1	346.64	-0.7	1.2	4.6	1.2
0	0	-2	2	-2	-182.62	-10.9	1.1	7.9	1.1
0	0	2	-2	2	182.62	11.4	1.1	-2.4	1.1
0	-1	-2	2	-2	-121.75	-4.4	1.1	-1.1	1.1
0	1	2	-2	2	121.75	2.4	1.1	1.7	1.1
1	0	0	-2	0	-31.81	-0.9	1.0	-4.8	1.0
-1	0	0	2	0	31.81	-5.3	1.0	-0.3	1.0
-1	0	0	0	0	-27.55	-15.3	1.1	-6.6	1.1
1	0	0	0	0	27.55	-0.1	1.1	1.8	1.1
-1	0	-2	2	-2	-23.94	2.1	1.1	-1.7	1.0
1	0	2	-2	2	23.94	-3.5	1.1	0.3	1.0
0	0	0	-2	0	-14.77	-1.4	1.0	7.3	1.0
0	0	0	2	0	14.77	1.7	1.0	1.2	1.0
-2	0	0	0	0	-13.78	-0.9	1.1	0.0	1.0
2	0	0	0	0	13.78	-2.3	1.1	-2.3	1.0
0	0	-2	0	-2	-13.66	-7.7	1.1	-12.0	1.1
0	0	2	0	2	13.66	-5.2	1.1	12.4	1.1
1	0	-2	-2	-2	-9.56	0.7	1.0	-2.3	1.0
-1	0	2	2	2	9.56	-0.1	1.0	-0.9	1.0
-1	0	-2	0	-2	-9.13	-4.5	1.1	1.7	1.1
1	0	2	0	2	9.13	-1.5	1.1	5.8	1.1
-1	0	-2	0	-1	-9.12	2.1	1.1	1.7	1.1
1	0	2	0	1	9.12	1.8	1.1	-5.3	1.1
0	0	-2	-2	-2	-7.10	-5.4	1.0	0.7	1.1
0	0	2	2	2	7.10	-2.6	1.0	0.4	1.1
-2	0	-2	0	-2	-6.86	-0.9	1.2	-1.5	1.2
2	0	2	0	2	6.86	-0.3	1.2	-1.2	1.2

long periods

short periods

http://ivsopar.obspm.fr

Use of optical observations for secular terms



Celestial pole offsets (from Vondrák 2012)
based on EOP catalog by Vondrák & Stefka, 2010, A&A, 509, A3

Use of LLR observations for long period terms

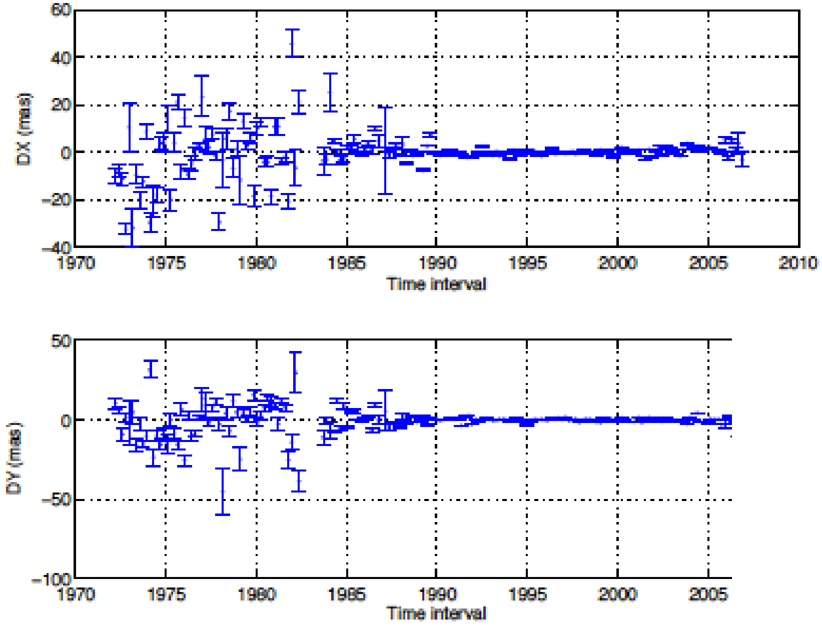


Fig. 5. Corrections to the celestial pole coordinates (*DX*, *DY*) with their formal errors using (70-day resolution).

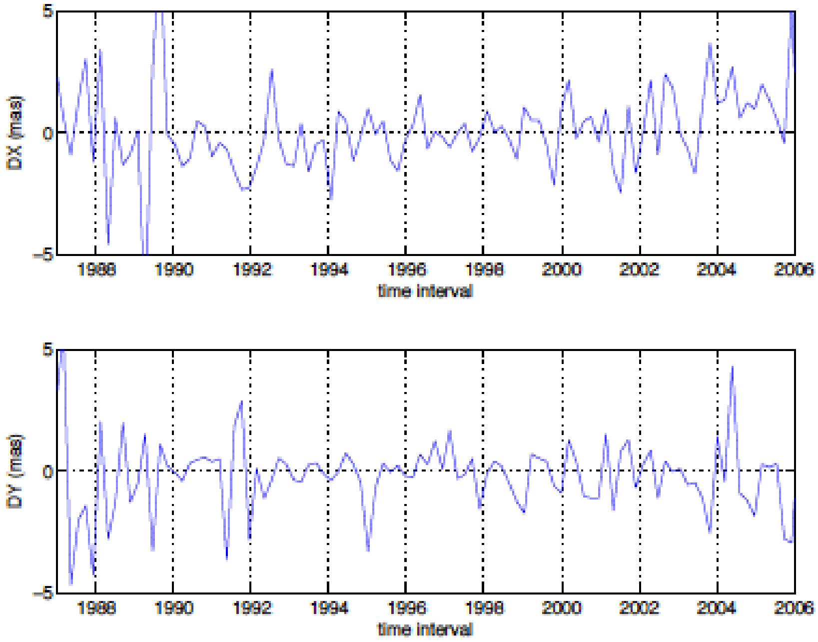


Fig. 6. Corrections to the celestial pole coordinates (*DX*, *DY*) using LLR observations over the period 1987-2006 (70-day resolution).

Zerhouni & Capitaine, 2009, A&A 507, 1687-1695

Use of GNSS observations for short period terms

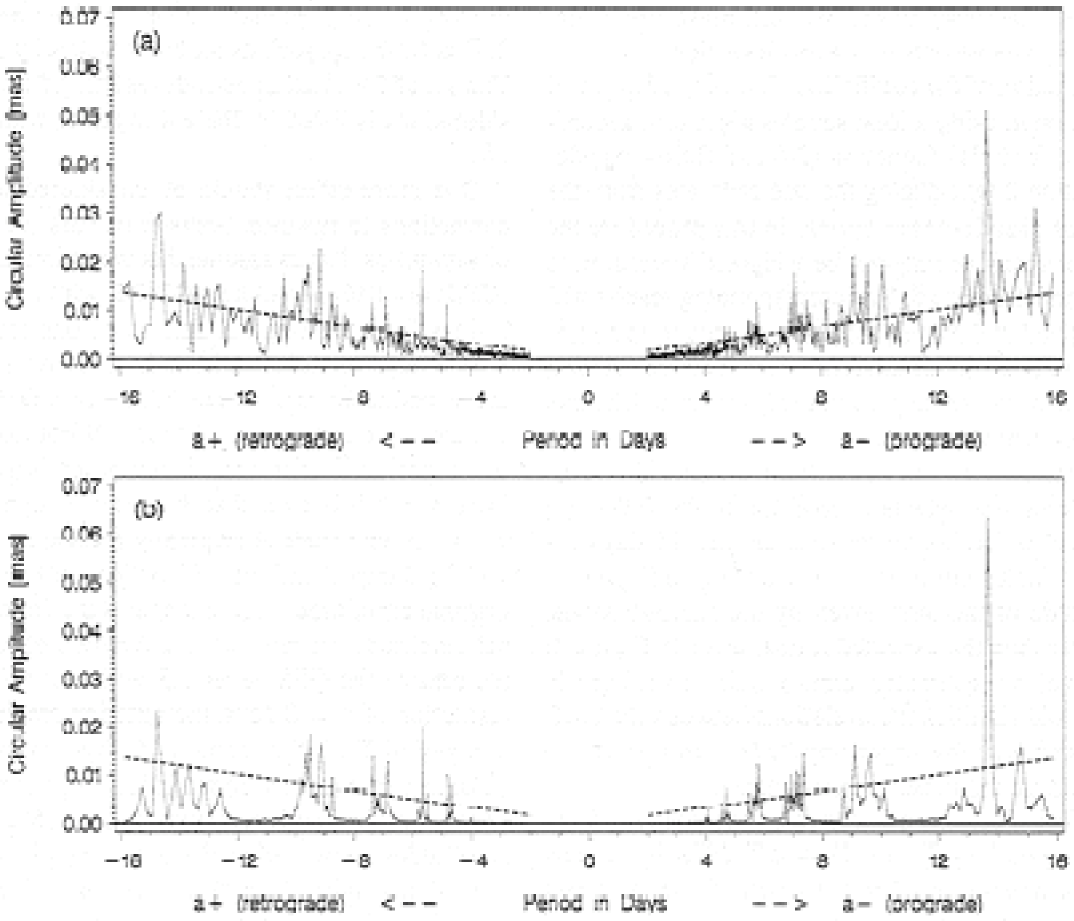


Figure 8. Spectrum of circular nutation amplitudes (see (40)) at low periods generated from (a) the N3 series of GPS nutation rates relative to the IAU80 model, converted to actual nutation amplitudes using (37), and (b) the differences between the IERS96 and the IAU80 model. The dashed lines indicate the 1σ uncertainties of the amplitudes as expected according to (33) (and (40)).

Ungoing work on « Nutation determination using the Global Positioning System » by Yao et al 2012: poster JD7-3-1372

project in cooperation with Vienna TU (R. Weber, E. Umnig)

GINS and Bernese software

Rothacher et al., JGR 1999, 104, 4835; also: Weber et al., 2001

Conclusions

- The IAU 2006/2000 precession/nutation has been shown to be accurate up to a few hundreds of μas/cy in the linear term, a few tens of μas for the 18.6-yr nutation and better than about 15 μas for the other terms.

- However, this results from VLBI comparisons only. Other techniques should be used for check. For the moment no one can provide additional information, but studies are being done for improving the situation.

- The discrepancies of the IAU model are at the limit of what further theoretical computations predict; It is not possible to discreminate between several predictions yet.